Involute Gear Tooth Contact And Bending Stress Analysis

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ABSTRACT

This paper presented analysis of Bending stress and Contact stress of Involute spur gear teeth in meshing. There are several kinds of stresses present in loaded and rotating gear teeth. Bending stress and contact stress (Hertz stress) calculation is the basic of stress analysis. It is difficult to get correct answer on gear tooth stress by implying fundamental stress equation, such as Lewis formula for bending stress and Hertz equation for contact stress. Various research methods such as Theoretical, Numerical and Experimental have been done throughout the years. This paper shows the theoratical and numeriacal approach to calculate bending and contact stress. The results where further compared with ANSYS result to validate.

KEYWORDS: *Spur Gear, Bending Stress, Contact Stress, ANSYS.*

I. INTRODUCTION

When one investigates actual gears in service, the conditions of the surface and bending failure are two of the most important features to be considered. The finite element method is very often used to analyze the stress states of elastic bodies with complicated geometries, such as gears. There are published papers, which have calculated the elastic stress distributions in gears.In these works, various calculation methods for the analysis of elastic contact problems have been presented. The finite element method for two-dimensional analysis is used very often. It is essential to use a three-dimensional analysis if gear pairs are under partial and non uniform contact. However, in the three dimensional calculation, a problem is created due to the large computer memory space that is necessary. In this chapter to get the gear contact stress a 2-D model was used. Because it is a nonlinear problem it is better to keep the number of nodes and elements as low as possible. In the bending stress analysis -D models are used for simulation.

II. ANALYTICAL PROCEDURE

From the results obtained in chapter 3 the present method is an effective and accurate method, which is proposed to estimate the tooth contact stresses of a gear pair.. Using the present method, the tooth contact stresses and the tooth deflections of a pair of spur gears analyzed by ANSYS 7.1 Since the present method is a general one, it is applicable to many types of gears. In early works, the following conditions were assumed in advance:

- There is no sliding in the contact zone between the two bodies
- The contact surface is continuous and smooth

Using the present method ANSYS can solve the contact problem and not be limited by the above two conditions. A two-dimensional and an asymmetric contact model were built. First, parameter definitions were given and then many points of the involute profile of the pinion and gear were calculated to plot an involute profile using a cylindrical system. The equations of an involute curve below were taken from Buckingham.

$$r = r_b * (1 + \beta^2)^{1/2}$$

$$\theta = \tan\phi - \phi = \mathrm{inv}\phi$$

where r = radius to the involute form, b r = radius of the base circle

$\beta = \varphi + \xi$

θ = vectorial angle at the pitch circle

ξ = vectorial angle at the top of the tooth

φ = pressure angle at the pitch circle

1 φ = pressure angle at radius r

One spur tooth profile was created using equation , shown in Figure 1, as are the outside diameter circle, the dedendum circle, and base circle of the gear. Secondly, in ANSYS from the tool bars using “CREATE”, “COPY”, “MOVE”, and “MESH” and so on, any number of teeth can be created and then kept as the pair of gear teeth in contact along the line of the action. The contact conditions of gear teeth are sensitive to the geometry of the contacting surfaces, which means that the element near the contact zone needs to be refined. It is not recommended to have a fine mesh everywhere in the model, in order to reduce the computational requirements. There are two ways to build the fine mesh near the contact surfaces. One is the same method as presented in chapter 3, a fine mesh of rectangular shapes were constructed only in the contact areas. The other one, “SMART SIZE” in ANSYS, was chosen and the fine mesh near the contact area was automatically created.

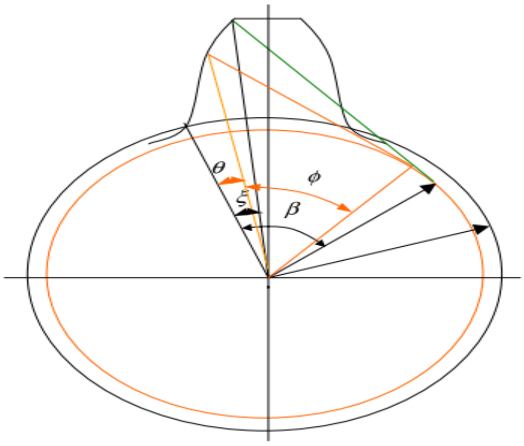

Figure 1 - Involutometry of a spur gear

III. ROTATION COMPATIBILITY OF THE GEAR BODY

In order to know how much load is applied on the contact stress model and the bending stress model, evaluating load sharing between meshing gears is necessary. It is also an important concept for transmission error. It is a complex process when more than one-tooth pair is simultaneously in contact taking into account the composite tooth deflections due to bending, shearing and contact deformation. This section presents a general approach as to how the load is shared between the meshing teeth in spur gear pairs. When the gears are put into mesh, the line tangent to both base circles is defined as the line of action for involute gears. In one complete tooth mesh circle, the contact starts at points A shown in Figure 2 where the outside diameter circle, the addendum circle of the gear intersects the line of action. The mesh cycle ends at point E, as shown in Figure 3 where the outside diameter of the pinion intersects the line of action.

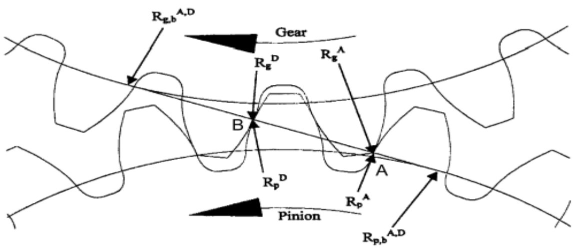

Figure 2 Illustration of one complete tooth meshing cycle

Consider two identical spur gears in mesh. When the first tooth pair is in contact at point A it is between the tooth tip of the output gear and the tooth root of the input gear (pinion). At the same time a second tooth pair is already in contact at point D in Figure 4.3. As the gear rotates, the point of contact will move along the line of action APE. When the first tooth pair reaches point B shown in Figure 4.4, the second tooth pair disengage at point E leaving only the first tooth pair in the single contact zone. After this time there is one pair of gear in contact until the third tooth pair achives in contact at point A again. When this tooth pair rotates to point D, the another tooth pair begins engagement at point A which starts another mesh cycle. After this time there are two pairs of gear in contact until the first tooth pair disengage at point

E. Finally, one complete tooth meshing cycle is completed when this tooth pair rotates to point E. To simplify the complexity of the problem, the load sharing compatibility condition is based on the assumption that the sum of the torque contributions of each meshing tooth pair must equal the total applied torque.

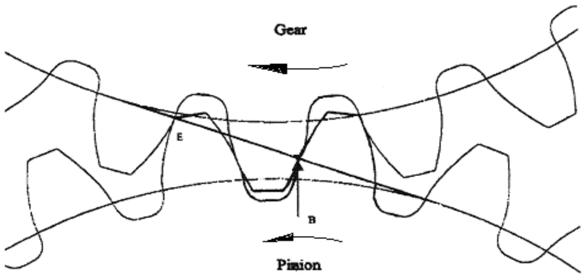

Figure 3 Different positions for one complete tooth meshing cycle

Analytical equations can also be developed for the rotation of the gear and pinion hubs, including the effects of tooth bending deflection and shearing displacement and contact deformation . In the pinion reference frame, it is assumed that the pinion hub remains stationary, while the gear rotates due to an applied torque. Considering the single pair contact zone at point B, the condition of angular rotation of the gear body will then be given by

For the pinion

For the gear

where and are the tooth displacement vectors caused by bending and shearing for pairs B of the pinion and gear respectively,

and are the contact deformation vectors of tooth pair B of the pinion and gear respectively.

denotes the transverse plane angular rotation of the pinion body caused by bending deflection, shearing displacement and contact deformation of the tooth pair B while the gear is stationary. Conversely, for the gear rotation while the pinion is stationary, gives the transverse plane angular rotations of the gear body.

IV. CALCULATION PART OF CONTACT STRESS, BENDING STRESS AND DEFORMATION.

Different Terms used in Calculation of Deformation and Bending Stress.
Velocity Factor between Gear and Pinion (K_V) = 1
Face width (F)= 40mm.
Module (m) = 01 (Between gear and pinion).
No. of teeth (x) , Gear=40 and Pinion=20
Pitch radius Pinion (r_{p1}) = 50mm
Pitch radius Gear (r_{p2}) = 95mm
Pressure Angle (ø) = 20^0
Constant (K_1) = 01
Constant (K_2) = 01
Total Load (W)= 1000N.
Poisson's Ratio (μ) = 0.3
y= 20mm.
Y=10mm.
Addendum of Gear (h_1) and Pinion (h_2)=10mm.
Young Modulus of Elasticity = 2 x 10^5
Tooth Thickness = 14.95mm.
Gear Ratio=2.
Bending Stress Calculation.

$$\sigma_b = \frac{W_t}{b \times m \times y}$$

Where,

$W_t =$

mm

$$\sigma_b = \frac{W_t}{K_v \times F \times m \times y}$$

$$\sigma_b = \frac{1000}{1 \times 40 \times 1 \times 26.66} = 0.93 \ N/mm^2$$

Contact Stress (Hertz Stress)

Width of the band of contact (B).

$$B = \sqrt{\frac{16 \times W\ (K1 + K2)R1 \times R2}{l\ (R1 + R2)}}$$

$$B = \sqrt{\frac{16 \times 1000\ (1 + 1)17.10 \times 32.49}{84.44(17.10 + 32.49)}} = 46.07mm$$

Where R1, R2= Radius of Curvature of an involutes curves at the contact points.
$R1 = r_{p1} Sin ø = 50 \sin 20 = 17.10$

$R2 = r_{p2} Sin ø = 95 \sin 20 = 32.49$

$$l^2 = \frac{2\ kWR}{\pi} = 7130.14mm \quad and\ l = 84.44mm$$

$$R = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{17.10} - \frac{1}{32.49}\right)^{-1} = 11.20mm$$

Maximum Contact Stress (

$$\sigma_c = \frac{4 \times W}{L \times \pi \times B}$$

$$\sigma_c = \frac{4 \times 1000}{84.44 \times \pi \times 46.07} = 0.32\ N/mm^2$$

To Determine Deformation (δ)

Coefficient of Compliance (C) .

$$C = \frac{E \times b \times \delta}{P}$$

$C = C_B + C_F + C_H$

Where:

C_B= Bending Compliance.

C_H= Hertz Compliance.

C_F= Root Compliance.

$$C_B = 12 \times cos^2\emptyset\left[I_1 + I_2\left\{0.2(1+\mu) + \frac{tan^2\emptyset}{12}\right\}\right]$$

$$I_1 = \int_0^y \frac{dy}{t} = \int_0^{20} \frac{20}{17}\,dy = 23.52$$

$$I_2 = \int_0^y \frac{(y-Y)^2}{t^3} \times dy = \int_0^{20} \frac{(10-20)^2}{17^3}\,dy = 0.40$$

$$C_B = 12 \times cos^2(20)\left[23.52 + 0.40\left\{0.2(1+0.3) + \frac{tan^2 20}{12}\right\}\right] = 65.78$$

$$C_H = \frac{2(1-\mu^2)}{\pi}\left[In\frac{4h_1h_2}{l^2} - \frac{\mu}{1-\mu}\right]$$

$$C_H = \frac{2(1-0.3^2)}{\pi}\left[In\frac{4\times 10\times 10}{7130.4} - \frac{0.3}{1-0.3}\right] = -0.95$$

Where

are the addendum of gear and pinion = 10mm.

$$C_F = (1-\mu)^2 cos^2\varphi\left[\frac{50}{3\pi}\left(\frac{y}{t_f}\right)^2 + \frac{2(1-2\mu)}{1-\mu}\times\frac{y}{t_f} + \frac{4.82}{\pi}\left(1+\frac{tan^2\varphi}{2.4(1+\mu)}\right)\right]$$

$$C_F = (1-0.3)^2 cos^2 20\left[\frac{50}{3\pi}\left(\frac{20}{18}\right)^2 + \frac{2(1-2\times 0.3)}{1-0.3}\times\frac{20}{18} + \frac{4.82}{\pi}\left(1+\frac{tan^2 20}{2.4(1+.3)}\right)\right]$$

$$C_F = 3.096$$

$$C = C_B + C_F + C_H$$

$$C = 65.78 + 3.096 + (-0.95) = 67.926$$

$$C = \frac{E \times b \times \delta}{P}$$

$$67.926 = \frac{2\times 10^5 \times 40 \times \delta}{1000}$$

Contact Stress, bending stress and deformation where further validate by FEM Analysis. Reference

FEM Deformation and Stress Analysis

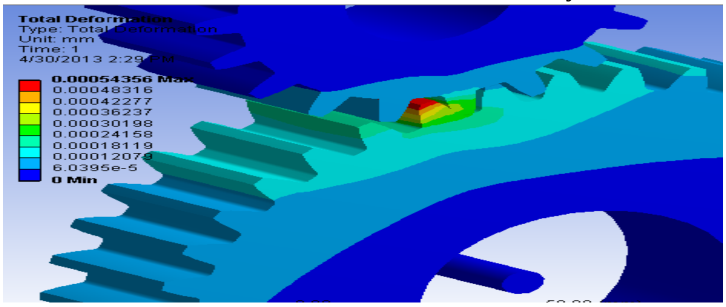

Fig-4 Deformation at 1 KN

Deformation Analysis: It has been found that deformation is maximum at the top land of face width. This indicates that top land area of spur gear is under high temperature and pressure due to which deformation is maximum at its top land. Here the deformation is 0.00054mm and calculated deformation is 0.00084mm. by compairing this two results, we can say that calculated deformation and Analytical deformation is validate.

Bending Sress Analysis:

It has been found that Stress is maximum at the contact surface of mating gear as shown in fig-5. The calculated bending is 0.93 MPa and contact stress is 0.32 MPa. So the calculated equivalent stress can be consider 1.25 MPa. But the Analytical Equivalent stress is found to be 4.416 MPa. So here the further modification is required to validate the results.

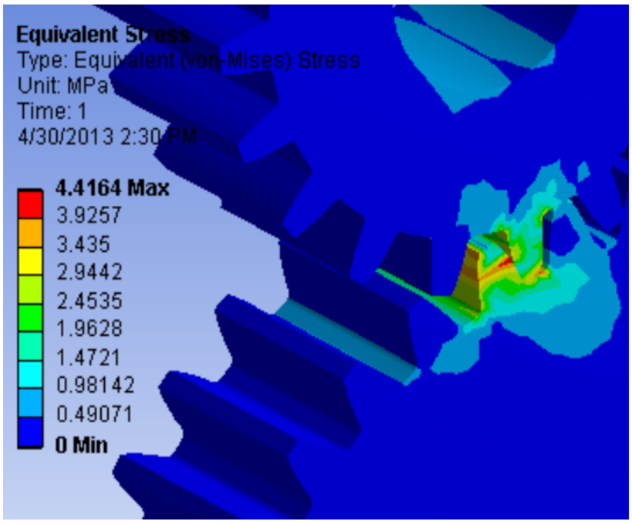

Fig 5 Stress at 1KN

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